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Design of State Estimator for a Class of Generalized Chaotic Systems

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ABSTRACT

In this paper, a class of generalized chaotic systems is considered and the state observation problem of such a system is investigated. Based on the time-domain approach with differential inequality, a simple state estimator for such generalized chaotic systems is developed to guarantee the global exponential stability of the resulting error system. Besides, the guaranteed exponential decay rate can be correctly estimated. Finally, several numerical simulations are given to show the effectiveness of the obtained result.

KEYWORDS: *Chaotic system, state estimator, generalized chaotic systems, exponential decay rate*

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1. INTRODUCTION

In recent years, various forms of chaotic systems have been extensively and intensively studied; see, for example, [1-15] and the references therein. The study of chaotic systems not only allows us to understand the chaotic characteristics, but also we can use the research results in various chaos applications; such as chaotic synchronization design and controller design of chaotic systems. As we know, chaotic signals are highly unpredictable and the initial values are highly sensitive to the output signal.

Due to the defects of the measuring instrument or the uncertainties of the system, not all state variables are just measurable for a nonlinear system. At this time, the design of the state estimator is very important and needs to be seriously faced. The state estimator has come to take its pride of place in system identification and filter theory. Besides, the state estimator design for the state reconstruction of dynamic systems with chaos is in general not as easy as that without chaos. Based on the above-mentioned reasons, the state estimator design of chaotic systems is quite meaningful and crucial.

In this paper, the state reconstructor for a class of the generalized chaotic systems is considered. Using the time-domain approach with differential inequality, a state estimator for such generalized chaotic systems will be provided to guarantee the global exponential stability of the resulting error system. In addition, the guaranteed exponential decay rate can be accurately estimated. Finally, some numerical examples will be given to illustrate the effectiveness of the obtained result.

2. PROBLEM FORMULATION AND MAIN RESULTS

In this paper, we consider the following generalized chaotic systems

$$\dot{x}_1(t)=ax_1(t)+bx_2(t) \tag{1a}$$

$$\dot{x}_2(t)=f_1(x_1(t),x_2(t),x_3(t)) \tag{1b}$$

$$\dot{x}_3(t)=cx_3(t)+f_2(x_1(t))\cdot f_3(x_2(t)), \tag{1c}$$

$$y(t)=\alpha x_1(t)+\beta x_2(t), \tag{1d}$$

where $x(t):=[x_1(t) \quad x_2(t) \quad x_3(t)]^T \in\Re^3$ is the state vector, $y(t)\in\Re$ is the system output, f_2 and f_3 are polynomials with $\deg(f_i)\geq 1, \forall\, i\in\{2,3\}$, and a,b,c are the parameters of state equation, with $c<0$ and $b\neq 0$. In addition, we select α and β, the parameters of output equation, satisfying $\frac{\alpha b}{\beta}>a-c$.

Remark 1: In fact, certain well-known chaotic systems, such as generalized Lorenz chaotic system [11], Lü chaotic system [12], generalized Chen chaotic system [13], unified chaotic system [14], and Pan chaotic system [15], are the special cases of the system (1).

A. System (1) is called the generalized Lorenz chaotic system [11] in case of

$$a=-10-25\Delta k,\quad b=10+25\Delta k,$$
$$c=\frac{-8-\Delta k}{3},\ \text{ with } 0\le\Delta k<0.8,$$
$$f_1=(28-35\Delta k)x_1+(29\Delta k-1)x_2-x_1x_3,$$
$$f_2=x_1,\ f_3=x_2.$$

B. System (1) is called the Lü chaotic system [12] in case of

$$a=-10-25\Delta k,\quad b=10+25\Delta k,$$
$$c=\frac{-8-\Delta k}{3},\ \text{ with } \Delta k=0.8,$$
$$f_1=(28-35\Delta k)x_1+(29\Delta k-1)x_2-x_1x_3,$$
$$f_2=x_1,\ f_3=x_2.$$

C. System (1) is called the generalized Chen chaotic system [13] in case of

$$a=-10-25\Delta k,\quad b=10+25\Delta k,$$
$$c=\frac{-8-\Delta k}{3},\ \text{ with } 0.8<\Delta k<1,$$
$$f_1=(28-35\Delta k)x_1+(29\Delta k-1)x_2-x_1x_3,$$
$$f_2=x_1,\ f_3=x_2.$$

D. System (1) is called the unified chaotic system [14] in case of

$$a=-10-25\Delta k,\quad b=10+25\Delta k,$$
$$c=\frac{-8-\Delta k}{3},\ \text{ with } 0\le\Delta k\le 1,$$
$$f_1=(28-35\Delta k)x_1+(29\Delta k-1)x_2-x_1x_3,$$
$$f_2=x_1,\ f_3=x_2.$$

E. System (1) is called the Pan chaotic system [15] in case of

$$a=-10,\ b=10,\ c=\frac{-8}{3},\ f_1=16x_1-x_1x_3,$$
$$f_2=x_1,\ f_3=x_2.$$

It is a well-known fact that since states are not always available for direct measurement, states must be estimated. The objective of this paper is to search a state estimator for the system (1) such that the global exponential stability of the resulting error systems can be guaranteed. In what follows, $\|x\|$ denotes the Euclidean norm of the column vector x and $|a|$ denotes the absolute value of a real number a.

Before presenting the main result, let us introduce a definition which will be used in the main theorem.

Definition 1: The system (1) is exponentially state reconstructible if there exist a state estimator $E\,\dot{z}(t)=h(z(t),y(t))$ and positive numbers k and η such that

$$\|e(t)\|:=\|x(t)-z(t)\|\le k\exp(-\eta t),\quad \forall t\ge 0,$$

where $z(t)$ expresses the reconstructed state of the system (1). In this case, the positive number η is called the exponential decay rate.

Now we present the main result.

Theorem 1: The system (1) is exponentially state reconstructible. Besides, a suitable state estimator is given by

$$\dot{z}_1(t)=\left(a-\frac{\alpha b}{\beta}\right)z_1(t)+by(t), \tag{2a}$$
$$z_2(t)=\frac{1}{\beta}y(t)-\frac{\alpha}{\beta}z_1(t), \tag{2b}$$
$$\dot{z}_3(t)=cz_3(t)+f_2(z_1(t))f_3(z_2(t)), \tag{2c}$$

with the guaranteed exponential decay rate $\eta:=-c$.

Proof. Define $\alpha_1:=\frac{\alpha b}{\beta}-a$, from (1), (2) with

$$e_i(t):=x_i(t)-z_i(t),\quad \forall i\in\{1,2,3\}, \tag{3}$$

it can be readily obtained that

$$\begin{aligned}\dot{e}_1(t)&=\dot{x}_1(t)-\dot{z}_1(t)\\&=ax_1(t)+bx_2(t)-\left(a-\frac{\alpha b}{\beta}\right)z_1(t)-by(t)\\&=ax_1(t)+b\left[\frac{y(t)-\alpha x_1(t)}{\beta}\right]\\&\quad-\left(a-\frac{\alpha b}{\beta}\right)z_1(t)-by(t)\\&=-\left(\frac{\alpha b}{\beta}-a\right)[x_1(t)-z_1(t)]\\&=-\alpha_1e_1(t),\quad \forall t\ge 0.\end{aligned}$$

It results that

$$\frac{d[e_1(t)\exp(\alpha_1t)]}{dt}=0$$
$$\Rightarrow\ [e_1(t)\exp(\alpha_1t)]=e_1(0)$$
$$\Rightarrow\ e_1(t)=\exp(-\alpha_1t)e_1(0)$$
$$\Rightarrow\ |e_1(t)|=|e_1(0)|\cdot\exp(-\alpha_1t),\quad \forall t\ge 0. \tag{4}$$

Moreover, form (1)-(4), we have

$$\begin{aligned}e_2(t)&=x_2(t)-z_2(t)\\&=\frac{y(t)-\alpha x_1(t)}{\beta}-\left[\frac{1}{\beta}y(t)-\frac{\alpha}{\beta}z_1(t)\right]\\&=\frac{-\alpha}{\beta}[x_1(t)-x_2(t)]\\&=\frac{-\alpha}{\beta}e_1(t)\\&=\frac{-\alpha}{\beta}\exp(-\alpha_1t)e_1(0),\quad \forall t\ge 0.\end{aligned}$$

Thus, one has

$$|e_2(t)|=\left|\frac{-\alpha}{\beta}\right||e_1(0)|\cdot\exp(-\alpha_1t),\quad \forall t\ge 0. \tag{5}$$

Define the polynomials $h_i(x,z):=\frac{f_i(x)-f_i(z)}{x-z},\quad \forall i\in\{2,3\}$, and form (1)-(5), it yields

$$\begin{aligned}\dot{e}_3(t)&=\dot{x}_3(t)-\dot{z}_3(t)\\&=c[x_3(t)-z_3(t)]+f_2(x_1(t))\cdot f_3(x_2(t))\\&\quad-f_2(z_1(t))f_3(z_2(t))\end{aligned}$$

$$
\begin{aligned}
&= ce_3(t) + f_3(x_2(t))\cdot[f_2(x_1(t)) - f_2(z_1(t))] \\
&\quad + f_2(z_1(t))\cdot[f_3(x_2(t)) - f_3(z_2(t))] \\
&= ce_3(t) + f_3(x_2(t))\cdot h_2(x_1(t), z_1(t)) \\
&\quad \cdot [x_1(t) - z_1(t)] \\
&\quad + f_2(z_1(t))\cdot h_3(x_2(t), z_2(t))[x_2(t) - z_2(t)] \\
&= ce_3(t) + f_3(x_2(t))\cdot h_2(x_1(t), z_1(t))e_1(t) \\
&\quad + f_2(z_1(t))\cdot h_3(x_2(t), z_2(t))e_2(t), \quad \forall\, t \ge 0.
\end{aligned}
$$

This implies that

$$
\begin{aligned}
\frac{d[e_3(t)\exp(-ct)]}{dt} &= f_3(x_2(t))\cdot h_2(x_1(t), z_1(t)) \\
&\quad \cdot e_1(t)\cdot \exp(-ct) \\
&\quad + f_2(z_1(t))\cdot h_3(x_2(t), z_2(t)) \\
&\quad \cdot e_2(t)\cdot \exp(-ct)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\ & e_3(t)\exp(-ct) - e_3(0) \\
&= \int_0^t f_3(x_2(t))\cdot h_2(x_1(t), z_1(t))e_1(t) \\
&\quad \cdot \exp(-ct)\,dt \\
&\quad + \int_0^t f_2(z_1(t))\cdot h_3(x_2(t), z_2(t))e_2(t) \\
&\quad \cdot \exp(-ct)\,dt
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\ & e_3(t)\exp(-ct) - e_3(0) \\
&\le M\int_0^t \exp(-\alpha_1 t - ct)\,dt \\
&= \frac{M}{-\alpha_1 - c}[-1 + \exp(-\alpha_1 t - ct)] \le \frac{M}{\alpha_1 + c}
\end{aligned}
$$

$$
\Rightarrow\ e_3(t) \le \left[e_3(0) + \frac{M}{\alpha_1 + c}\right]\cdot \exp(ct), \quad \forall\, t \ge 0, \tag{6}
$$

where

$$
\begin{aligned}
M &\ge |f_3(x_2(t))\cdot h_2(x_1(t), z_1(t))|\cdot|e_1(0)| \\
&\quad + |f_2(z_1(t))\cdot h_3(x_2(t), z_2(t))|\cdot\left|\frac{-\alpha}{\beta}\right||e_1(0)|.
\end{aligned}
$$

Consequently, by (4)-(6), we conclude that

$$
\|e(t)\| = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \le k\cdot\exp(ct), \quad \forall\, t \ge 0,
$$

with $k = \max\left\{|e_1(0)|, \left|\frac{-\alpha}{\beta}\right||e_1(0)|, \left|e_3(0) + \frac{M}{\alpha_1 + c}\right|\right\}$. This completes the proof. □

3. NUMERICAL SIMULATIONS

Example 1: Consider the following generalized Lorenz chaotic system:

$$\dot{x}_1(t) = -27.5x_1(t) + 27.5x_2(t), \tag{7a}$$
$$\dot{x}_2(t) = 3.5x_1(t) + 19.3x_1(t) - x_1(t)x_3(t), \tag{7b}$$
$$\dot{x}_3(t) = -2.9x_3(t) + x_1(t)x_2(t), \tag{7c}$$
$$y(t) = x_1(t) + x_2(t). \tag{7d}$$

Comparison of (7) with (1), one has $a = -27.5, b = 27.5, c = -2.9$, $\alpha = \beta = 1$, $f_1(x_1, x_2, x_3) = 3.5x_1 + 19.3x_1 - x_1x_3$, $f_2(x_1) = x_1$, and $f_3(x_2) = x_2$. By Theorem 1, we conclude that the system (7) is exponentially state reconstructible by the state estimator

$$\dot{z}_1(t) = -54z_1(t) + 27.5y(t), \tag{8a}$$
$$z_2(t) = y(t) - z_1(t), \tag{8b}$$
$$\dot{z}_3(t) = -2.9z_3(t) + z_1(t)z_2(t), \tag{8c}$$

with the guaranteed exponential decay rate $\eta := 2.9$. The typical state trajectories of (7) and the time response of error states for the systems (7) and (8) are depicted in Figure 1 and Figure 2, respectively.

Example 2: Consider the following Pan chaotic system:

$$\dot{x}_1(t) = -10x_1(t) + 10x_2(t), \tag{9a}$$
$$\dot{x}_2(t) = 16x_1(t) - x_1(t)x_3(t), \tag{9b}$$
$$\dot{x}_3(t) = \frac{-8}{3}x_3(t) + x_1(t)x_2(t), \tag{9c}$$
$$y(t) = x_1(t) + x_2(t). \tag{9d}$$

Comparison of (9) with (1), one has $a = -10, b = 10, c = \frac{-8}{3}$, $\alpha = \beta = 1$, $f_1(x_1, x_2, x_3) = 16x_1 - x_1x_3$, $f_2(x_1) = x_1$, and $f_3(x_2) = x_2$. By Theorem 1, we conclude that the system (9) is exponentially state reconstructible by the state estimator

$$\dot{z}_1(t) = -20z_1(t) + 10y(t), \tag{10a}$$
$$z_2(t) = y(t) - z_1(t), \tag{10b}$$
$$\dot{z}_3(t) = \frac{-8}{3}z_3(t) + z_1(t)z_2(t), \tag{10c}$$

with the guaranteed exponential decay rate $\eta := \frac{8}{3}$. The typical state trajectories of (9) and the time response of error states for the systems (9) and (10) are depicted in Figure 3 and Figure 4, respectively.

4. CONCLUSION

In this paper, a class of generalized chaotic systems has been considered and the state observation problem of such systems has been investigated. Based on the time-domain approach with differential inequality, a simple state estimator for such generalized chaotic systems has been developed to guarantee the global exponential stability of the resulting error system. Besides, the guaranteed exponential decay rate can be correctly estimated. Finally, several numerical simulations have been proposed to show the effectiveness of the obtained result.

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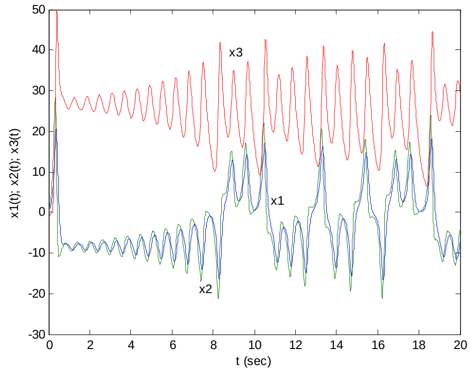

Figure 1: Typical state trajectories of the system (7).

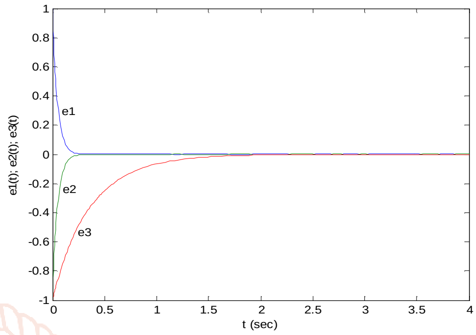

Figure 2: The time response of error states, with $e_i = x_i - z_i, \forall i \in \{1,2,3\}$.

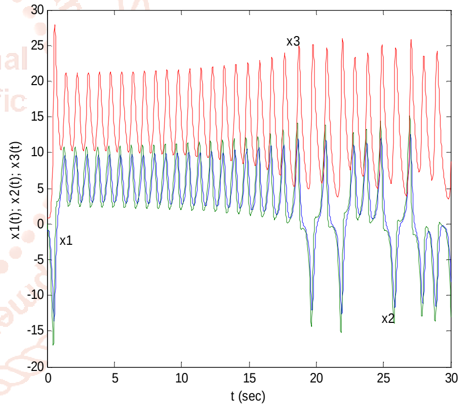

Figure 3: Typical state trajectories of the system (9).

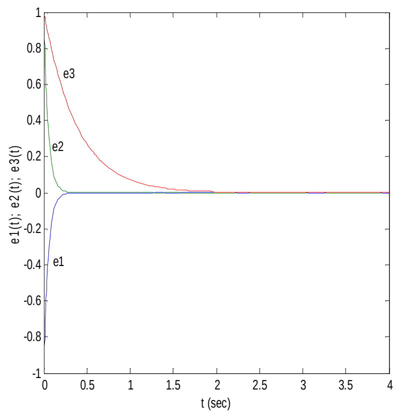

Figure 4: The time response of error states, with $e_i = x_i - z_i, \forall i \in \{1,2,3\}$.